Due June 24, 11:59pm on Gradescope.

The following are warm-up exercises and are *not* to be turned in. You may treat these as extra practice problems.

1.1.25, 1.5.25, 1.7.11, 1.7.12, 1.7.27, 1.7.40, 1.8.10, 1.8.16, 1.8.32, 2.1.40, 2.1.41, 2.2.21, 2.2.32, 2.2.38.

Turn in the following exercises. Remember to carefully justify every statement that you write, and to follow the style of proper mathematical writing. You may cite any result proved in the textbook or lecture, unless otherwise mentioned. Each problem is worth 10 points, unless otherwise mentioned.

1. 1.1.30(a, b).

2. 1.5.28(a, c, e, f, h). Justify your answer for each part.

3. 1.6.16. A one-sentence explanation for each part is sufficient.

4. Prove that $\sqrt{2} + \sqrt{3}$ is irrational. [Hint: you may assume without proof that $\sqrt{6}$ is irrational.]

5. 1.8.38. Here, "between" means *strictly* between.

6. Consider the subset $S = \{(x, y) : x^2 + y^2 \leq 1\}$ of $\mathbf{R} \times \mathbf{R}$. Do there exist subsets $A, B \subseteq \mathbf{R}$ such that $S = A \times B$?

7. 2.2.42. You may assume the result of 2.2.41; i.e. that $A \oplus B = (A \cup B) - (A \cap B)$.

8. For $n \geq 2$, let A_n be the subset $(1/n, 1)$ of the real numbers $\mathbf{R}$. Show that for any positive integer $k \geq 2$,
$$\bigcup_{n=2}^{k} A_n \neq (0, 1),$$
but also show that
$$\bigcup_{n=2}^{\infty} A_n = (0, 1).$$
[Hint: you may use the *archimedean property* of the real numbers: that is, for any two positive real numbers a and b, there is an integer n such that $na > b$.]

9. **(Bonus problem, 10 points)** Suppose the integers $1, 2, \ldots, 1002$ are written on a board. We may perform the following operation: select any two integers a and b on the board, erase them both, and replace them by writing the integer $(a + b)^2$ on the board. After performing this operation 1001 times, we are left with a single integer k written on the board. Show that k must be odd. You may assume basic facts about parity without proof; e.g. that the square of an odd integer is odd, that two odd integers sum to an even integer, etc. [Hint: think about the sum of the numbers written on the board.]